Experimental moments of the nucleon structure function F_2

M. Osipenko[a], W. Melnitchouk[b], S. Simula[c], S. Kulagin[d], G. Ricco[e] and CLAS Collaboration

[a]Istituto Nazionale di Fisica Nucleare, Sezione di Genova, Genoa, Italy 16146

[b]Jefferson Lab, Newport News, Virginia 23606

[c]Istituto Nazionale di Fisica Nucleare, Sezione Roma III, Roma, Italy 00146

[d]Institute for Nuclear Research of Russian Academy of Science, Moscow, Russia 117312

[e]Università di Genova, Genoa, Italy 16146

Experimental data on the F_2 structure functions of the proton and deuteron, including recent results from CLAS at Jefferson Lab, have been used to construct their $n \leq 12$ moments. A comprehensive analysis of the moments in terms of the operator product expansion has been performed to separate the moments into leading and higher twist contributions. Particular attention was paid to the issue of nuclear corrections in the deuteron, when extracting the neutron moments from data. The difference between the proton and neutron moments was compared directly with lattice QCD simulations. Combining leading twist moments of the neutron and proton we found the d/u ratio at $x \to 1$ approaching 0, although the precision of the data did not allow to exclude the 1/5 value. The higher twist components of the proton and neutron moments suggest that multi-parton correlations are isospin independent.

The Operator Product Expansion (OPE) is a powerful tool in QCD which allows measurable moments of hadronic structure functions to be related to series expansions of the moments in terms of twists. The first term in the series, corresponding to Leading Twist (LT), reflects the physics of asymptotic freedom, and is determined by single-parton distributions in the hadron. Subsequent terms in the series, or Higher Twists (HT), describe interactions between partons, or multi-parton correlations. The determination of the HTs is considerably more challenging, both experimentally and theoretically.

We analyzed data on experimentally extracted moments of the proton and deuteron structure functions F_2 [1] to separate LT and HT terms. The n-th moment of the F_2 structure function, including LT and HT contributions, can be written:

$$M_n(Q^2) = LT_n(\alpha_S) + \sum_{\tau=4,6} a_n^\tau \left(\frac{\alpha_S(Q^2)}{\alpha_S(\mu^2)}\right)^{\gamma_n^\tau} \left(\frac{\mu^2}{Q^2}\right)^{\frac{\tau-2}{2}} , \quad (1)$$

where LT_n is the leading, twist-2 moment, α_S is the running coupling constant, μ^2 is an arbitrary scale (taken to be 10 (GeV/c)^2), a_n^τ is the matrix element of corresponding QCD operators, γ_n^τ is the anomalous dimension and τ is the order of the twist.

The separation of the LT from the complete series is to some extent dependent on the order to which one calculates the LT Q^2-evolution. In Fig. 1 we compare the $n = 8$ moment of the LT term calculated at fixed orders in pQCD: Leading Order (LO), Next-to-Leading Order (NLO) and Next-to-Next-to-Leading Order (NNLO); and by using resummation of soft gluon emission (SGR) [2]: Leading Log (LL) and Next-to-Leading Log (NLL). The fixed order calculations appear to converge at NLO[1]. However, at fixed pQCD order the logarithmic precision of the LT term deteriorates the closer one gets to $x = 1$. Applying the SGR [2] we can improve the accuracy of the LT term in the large-x region. The

[1]The difference between NLO and NNLO is small with respect to uncertainties of the data

resummed LT calculated at NLL deviates significantly from the LL result, similarly to the NLO and LO at fixed order. Unfortunately, next-to-next-to-leading log (NNLL) calculations are not yet available to confirm that also resummed moments converge at second order. Nevertheless, the LT term is calculated to the best accuracy currently available.

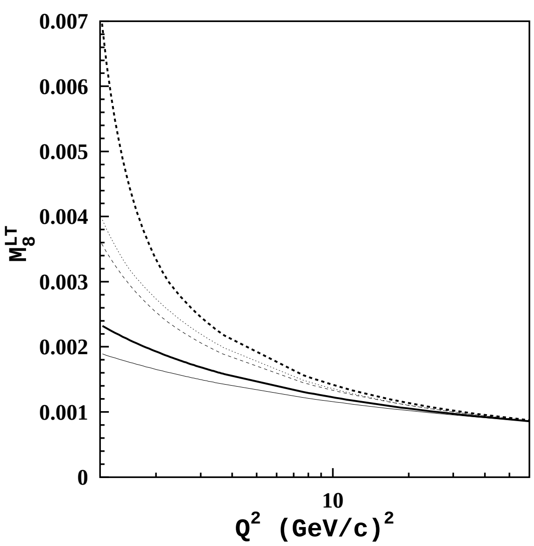

Figure 1. $n = 8$ LT moment calculated to different pQCD accuracy: thin solid line - LO, thin dashed line - NLO, thin dotted line - NNLO, thick solid line - LL thick dashed line NLL.

The extracted LT components of the proton and deuteron moments can be combined to form moments of the neutron F_2 structure function. In the nuclear Impulse Approximation (IA), the nuclear structure function can be written as a convolution of the nucleon structure function and a nucleon distribution function, f^D, in the deuteron. In moment space this translates into a product of moments, so that the neutron moments can be obtained from:

$$M_n^n(Q^2) = \frac{2M_n^D(Q^2)}{N_n^D} - M_n^p(Q^2) \ , \tag{2}$$

where M_n^p, M_n^n and M_n^D are the proton, neutron and deuteron moments, respectively, and N_n^D is the moment of the function f^D. The distribution function f^D was calculated from various deuteron wave functions [3].

The extracted LT proton and neutron moments can be combined to form Non-Singlet (NS) moments of the nucleon F_2 structure function, which can then be compared to lattice QCD simulations. A comparison of the extracted moments with recent lattice results from several groups is shown in Fig. 2. While a linear extrapolation of the lattice results to the physical pion mass overestimates our data significantly for $n = 2$ and 4, the results extrapolated using chiral effective [6] theory agree very well with our data. The data for higher moments are also of high precision, and it would be of considerable interest to compare these with higher lattice moments, especially since the effects of chiral loops are expected to be suppressed in the large-n (large-x) domain.

Another interesting result that can be obtained from the proton and neutron moments is related to the behavior of u and d quarks in the proton in the $x \to 1$ limit. At leading twist, the d/u ratio at large x can be extracted directly from the ratio of the neutron to proton structure functions F_2^n/F_2^p, which is in turn related to the ratio of the moments M_n^n/M_n^p for large n. Indeed, for $n \gg 1$ one finds that $M_n^n/M_n^p(n \to \infty) = F_2^n/F_2^p(x \to 1)$.

The n/p structure function moment ratios are shown in Fig. 3. Also indicated on the vertical axes are model predictions for the $x \to 1$ limits, namely the "standard" 1/4 value used in most of parton distribution fits (corresponding to a vanishing d/u ratio), and the value 3/7 expected from helicity conservation model (see Ref. [7]). The trend of our data is towards the lower value of the model predictions with increasing n, although the precision of the data does not exclude the higher value.

Indeed, at large n, both the data and the theoretical framework become more problematic,

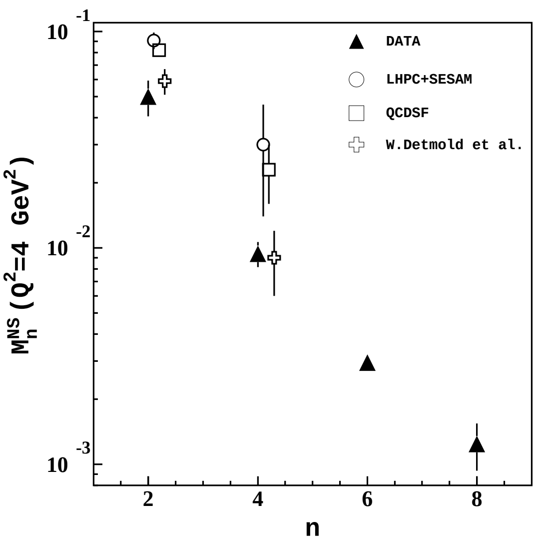

Figure 2. Moments of the non-singlet F_2 structure function compared with several lattice QCD simulations: filled triangles - present analysis, open circles - lattice simulations from Ref.[4] with linear extrapolation, open squares - lattice simulations from Ref.[5] with linear extrapolation, open crosses - lattice simulations from Ref.[5] with chiral extrapolation from Ref.[6].

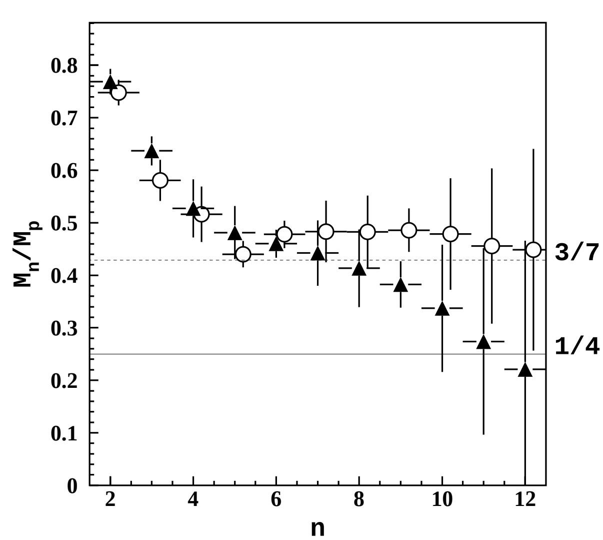

Figure 3. Ratio of neutron to proton moments as a function of n: filled triangles - this analysis, open circles - ratio without nuclear corrections applied. The solid (dashed) line indicates the scenario where $d/u \to 0(1/5)$ for $x \to 1$ [7].

making it more difficult to distinguish between the different hypotheses. From Fig. 3 one can also see the impact of the nuclear corrections on the deuteron moments. This is particularly evident for large n: for $n = 12$, which corresponds to x values of around 0.75, the nuclear correction introduces a factor of ~ 2.

Once the LT contribution to moments is determined, one can then study the isospin dependence of the HT contribution. The HTs provide important information about multiparton correlations inside the nucleon. We assume that final state interactions and meson exchange currents (in particular, the $1/Q^2$ components) is negligible above $Q^2 = 1$ (GeV/c)^2, so that the same nuclear corrections can be applied to the HTs as for the LT, Eq. 2. The total HT contributions to proton moments and the corrected deuteron moments can then be compared, Fig. 4. This comparison indicates that the total HT contribution is independent of isospin. The isovector combinations $p-n$ of structure functions F_2 should therefore be free of HT contributions, within the presented uncertainties.

In summary, we have analyzed experimental data on proton and deuteron F_2 structure function in order to extract their moments, and performed an OPE analysis to separate leading and higher twist contributions. By combining proton and deuteron moments and applying nuclear corrections, we extracted moments of the neutron F_2 structure function, paying particular attention to the issue of nuclear effects in the deuteron, which

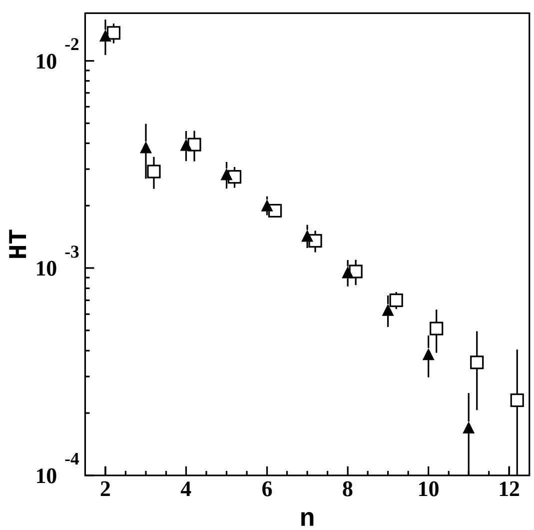

Figure 4. Total higher twist contribution to the proton (filled triangles) and deuteron (open squares) moments at $Q^2 = 2$ (GeV/c)^2.

are increasingly important for higher moments.

The LT part of the non-singlet moments were then obtained and related to lattice QCD moments available for $n = 2$ and 4. For large n, the ratio of the neutron to proton moments can be related to the ratio of u and d quark contributions in the proton in the $x \to 1$ limit. The HT contribution is related to the physics beyond the asymptotically free regime — namely, multiparton correlations. The results of our analysis can be summarized as follows:

- the ratio of neutron to proton moments is consistent with $F_2^n/F_2^p \to 1/4$ as $x \to 1$, although one cannot exclude the higher value of 3/7 suggested by helicity conservation arguments;

- the non-singlet moments are in excellent agreement with the lattice data [4,5], if these are extrapolated to physical quark masses taking into account chiral loops associated with the pion cloud [6], but underestimate the lattice results when linearly extrapolations are used;

- the total contribution of HTs is found to be isospin independent, which implies that in the isovector combination $p - n$ of F_2 structure functions the HTs are consistent with zero.

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